# A COURSE SYLLABUS – DOCTORAL SCHOOL

## REGARDING THE QUALIFICATION CYCLE FROM 2023TO 2027

| GENERAL INFORMATION ABOUT COURSE | |
|---|---|
| Course title | **Topology** |
| Name of the unit running the course | **Doctoral School at University of Rzeszów** |
| Type of course *(obligatory, optional)* | optional |
| Year and semester of studies | I/ II |
| Discipline | Mathematics |
| Language of Course | Polish |
| Name of Course coordinator | Prof. dr hab. Mykhaylo Zarichnyy |
| Name of Course lecturer | Prof. dr hab. Mykhaylo Zarichnyy |
| Prerequisites | Topology, Algebra, Introduction to Logic and Set Theory, I and II degree of studies |

**BRIEF DESCRIPTION OF COURSE**
**(100-200 words)**

The course is devoted to selected topics in topology, namely ultrametric spaces, their categorical descriptions and isometry groups, Borel classification of sets in topological (metric) spaces, concepts of combinatorial group theory (free groups, subgroups, free products) as well as application of geometric and topological methods to abstract group theory.

**COURSE LEARNING OUTCOMES AND METHODS OF EVALUATING LEARNING OUTCOMES**

| Learning outcome | The description of the learning outcome defined for the course | Relation to the degree programme outcomes (symbol) | Learning Format (Lectures, classes,…) | Method of assessment of learning outcomes (e.g. test, oral exam, written exam, project,…) |
|---|---|---|---|---|
| **Knowledge (no.)** | **(Knows and understands)** | | | |
| 1. | Modern publications concerning ultrametric and other spaces | P8S_WG1 | Lectures | Written exam |
| 2. | Directions of development of topology | P8S_WG2 | Lectures | Written exam |
| 3. | System of notions characteristic for topology | P8S_WG3 | Lectures | Written exam |
| 4. | Methodology of scientific research using interdisciplinary research techniques and tools | P8S_WK1 | Lectures | Written exam |
| **Skills (no.)** | **(Able to)** | | | |
| 1. | Use knowledge from various disciplines to analyze a scientific problem and apply appropriate techniques to solve it. | P8S_UW1 | Lectures | Written exam |
| 2. | Use scientific publications to solve problems. | P8S_UW2 | Lectures | Written exam |
| 3. | Make a critical analysis and evaluation of the results of scientific research. | P8S_UW3 | Lectures | Written exam |

| | | | | |
|---|---|---|---|---|
| 4. | Participate in international scientific discourse. | P8S_UK6 | Lectures | Written exam |
| **Social competence (no.)** | **(Ready to)** | | | |
| 1. | Recognizes the importance of knowledge in solving cognitive and practical problems | P8S_KK3 | Lectures | Written exam |

**LEARNING FORMAT – NUMBER OF HOURS**

| Semester (no.) | Lectures | Seminars | Lab classes | Internships | others | ECTS |
|---|---|---|---|---|---|---|
| 2 | 15 | | | | | 2 |

**METHODS OF INSTRUCTION**

Lectures, discussions

**COURSE CONTENT**

Ultrametric spaces, universal ultrametric spaces, their isometry groups.
Categorical methods in theory of ultrametric spaces.
Ultrametric normed spaces.
Descriptive set theory.
Polish spaces, universal spaces.
Borel sets, Borel hierarchy.
Baire property, meagre sets.
Relation to Borel hierarchy.
Analytic and coanalytic sets.
Projective sets and projective hierarchy.
Combinatorial group theory.
Generators and relations.
Free groups.
Graph of a group.
Presentation of subgroups.
The Reidemeister-Schreier method.
Free products with amalgamated subgroups.
Trees and free groups.

**COURSE ASSESSMENT CRITERIA**

The condition for passing the written exam is to obtain at least 50% of the points. The final grade is then determined according to the scale:

below 50% points – insufficient,

[50 – 60%) pts. – sufficient,

[60 – 70%) pts. – sufficient plus,

[70 – 80%) pts. - Good,

[80 – 90%) pts. – plus good,

[90 – 100%] pts. - very good.

**TOTAL PhD STUDENT WORKLOAD REQUIRED TO ACHIEVE THE INTENDED LEARNING OUTCOMES – NUMBER OF HOURS AND ECTS CREDITS**

| Activity | Number of hours |
|---|---|
| Scheduled course contact hours | 15 |
| Other contact hours involving the teacher (consultation hours, examinations) | 0 |
| Non-contact hours – student`s own work (preparation for classes or examinations, project, etc.) | 0 |
| **Total number of hours** | **15** |
| **Total number of ECTS credits** | **2** |

| **INSTRUCTIONAL MATERIALS** | |
|---|---|
| Compulsory literature: | Kechris, Alexander S. (1994). Classical Descriptive Set Theory. Springer-Verlag. ISBN 0-387-94374-9.<br>Willard, Stephen (2004) [1970]. General Topology. Mineola, N.Y.: Dover Publications. ISBN 978-0-486-43479-7.<br>W. Magnus, A. Karrass and D. Solitar, "Combinatorial Group Theory", Dover (1976).<br>Roger C. Lyndon; Paul E. Schupp. Combinatorial group theory I -Reprint of the 1977 ed. - Berlin; Heidelberg; New York; New York; Barcelona; Hong Kong; London; Milan; Paris; Singapore; Tokyo: Springer. 2001<br>ISBN 3-54G-41158-5 |
| Complementary literature: | Bestvina M. R-trees in topology, geometry, and group theory. In R.J. Daverman, R.B. Sher (eds.) Handbook of geometric topology, 55-91, North Holland, Amsterdam, 2002.<br>Oxtoby, John C. (1980), Measure and Category, Graduate Texts in Mathematics, vol. 2 (2nd ed.), Springer-Verlag, pp. 19–21, ISBN 978-0-387-90508-2.<br>Kaplansky, I. (1977), Set Theory and Metric Spaces, AMS Chelsea Publishing, ISBN 978-0-8218-2694-2.<br>Bruce Hughes, Trees and ultrametric spaces: a categorical equivalence, Advances in Mathematics, Volume 189, Issue 1, 2004, 148-191.<br>J.-P. Serre, Trees, Springer, 1980. |