**SYLLABUS**

**regarding the qualification cycle FROM 2023TO 2024**

1. Basic Course/Module Information

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| --- | --- |
| Course/Module title | Linear algebra with geometry (1 and 2) |
| Course/Module code \* |  |
| Faculty (name of the unit offering the field of study) | *College of Natural Sciences* |
| Name of the unit running the course | *Institute of Mathematics* |
| Field of study | Mathematics |
| Qualification level | First degree |
| Profile | *Academic* |
| Study mode | *Full-time* |
| Year and semester of studies | *1 Year, 1 and 2 semester* |
| Course type | *Basic* |
| Language of instruction | English |
| Coordinator | Ewa Rak, PhD |
| Course instructor | *Stanisława Kanas, PhD, DSc* |

\* - as agreed at the faculty

1.1.Learning format – number of hours and ECTS credits

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Semester  (n0.) | Lectures | Classes | Colloquia | Lab classes | Seminars | Practical classes | Internships | others | **ECTS credits** |
| 1 |  | 30 |  |  |  |  |  |  | 5 |
| 2 |  | 30 |  |  |  |  |  |  | 5 |

1.2. Course delivery methods

☒ conducted in a traditional way

☒ involving distance education methods and techniques

1.3. Course/Module assessment (exam, pass with a grade, pass without a grade)

Exam

2. Prerequisites

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| Basic knowledge of mathematics on secondary school level, Secondary-school certificate |

3. Objectives, Learning Outcomes, Course Content, and Instructional Methods

3.1. Course/Module objectives

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| --- | --- |
| O1 | Introduction to basic concepts of linear algebra. |
| O2 | Introduction to basic concepts of analytic geometry. |
| O3 | Introduction to the basic methods of proof used in linear algebra and analytical geometry. |
| O4 | Introduction to basic computational techniques used in linear algebra and analytical geometry. |

3.2. Course/Module Learning Outcomes (to be completed by the coordinator)

|  |  |  |
| --- | --- | --- |
| Learning Outcome | The description of the learning outcome  defined for the course/module | Relation to the degree programme outcomes |
| LO\_01 | defines classic concepts and formulates basic theorems in the field of linear algebra and analytical geometry | K\_W01, K\_W02,  K\_W03, K\_W04 |
| LO\_02 | has knowledge of the methods of proofs used in linear algebra and analytic geometry | K\_W02, K\_W03 |
| LO\_o3 | has knowledge of computational techniques used in linear algebra and analytical geometry | K\_W05 |
| LO\_04 | proves the basic theorems in the field of linear algebra and analytical geometry | K\_U02 |
| LO\_05 | uses the concepts of: group, field, linear space, vector, matrix, linear transformation, straight line and plane, conic section, rotational surface | K\_U12, K\_U13, K\_U14 |
| LO\_06 | is able to perform operations on complex numbers (in various forms) and interpret various sets of complex numbers, solves equations in the set of complex numbers | K\_U01, K\_U12 |
| LO\_07 | knows how to calculate determinants and knows their properties | K\_U01, K\_U12 |
| LO\_08 | solves systems of linear equations with constant coefficients, can use the geometric interpretation of solutions | K\_U01, K\_U12 |
| LO\_09 | finds linear transformation matrices at various bases | K\_U01, K\_U12 |
| LO\_10 | calculates the eigenvalues and eigenvectors of the matrix, is able to explain the geometric meaning of these concepts | K\_U01, K\_U12 |
| LO\_11 | can write different equations of lines and planes and examine their mutual position | K\_U01, K\_U14 |
| LO\_12 | uses the properties of conic curves in solving problems in solving problems | K\_U01, K\_U14 |
| LO\_13 | can write rotational surface equations | K\_U01, K\_U14 |
| LO\_14 | independently searches for information in the literature and correctly applies it | K\_U26 |
| LO\_15 | can formulate opinions on basic issues of linear algebra and analytical geometry | K\_K02 |
| LO\_16 | knows the limitations of their own knowledge and understands the need for further education | K\_K01 |

**3.3. Course content (to be completed by the coordinator)**

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| Content outline |
| **Operations**: basic properties and examples. **Algebraic structures and homomorphisms**: review of basic algebraic structures: groups, fields, homomorphisms of groups. **The field of complex numbers**: algebraic, trigonometric and exponential form of a complex number, operations on complex numbers, de Moivre's formula, roots of a complex number, geometric interpretations of sets of complex numbers. **Linear spaces**: linear combination of vectors, linear dependence and independence of vectors, linear space base, dimension of linear space, linear subspace, simple sum of linear subspaces, quotient space. |
| **Matrices**: basic concepts, various types of matrices, operations on matrices. **Determinant of the square matrix**: definition of the determinant, properties of determinants, methods of calculating determinants, Laplace formula, inverse matrix, minors and the rank of a matrix. **Systems of linear equations**: Kronecker-Capelli theorem, general form of solutions of systems of linear equations, study of the system of equations, Cramer's formulas. |
| **Line mappings**: definition of a linear mapping, kernel and image of a linear mapping, order of a linear mapping, monomorphism, epimorphism, isomorphism.  **Matrix representation of a linear mapping**: matrix multiplication and assembly of linear mappings; transition matrix, linear mapping matrix after base change. **Endomorphisms**: eigenvalue and eigenvector of endomorphism, characteristic polynomial. **Quadratic forms**: bilinear mapping, matrix and order of bilinear mapping, diagonalization of quadratic form. |
| **Euclidean vector spaces**: scalar product, norm determined by scalar product, Schwarz inequality, orthonormal basis, orthogonal matrix. **Vectors**: operations on vectors in R3: addition, subtraction, multiplication by scalar, scalar product, vector product. |
| **Analytical geometry in R2**: lines on a plane; definitions and equations of conic curves. **Analytical geometry in R3**: lines and a plane in space; rotating surfaces in R3 (cylinders, cones, hyperboloids, paraboloids). |

3.4. Methods of Instruction

e.g.

*Lecture: a problem-solving lecture/a lecture supported by a multimedia presentation/ distance learning*

*Classes: text analysis and discussion/project work (research project, implementation project, practical project)/ group work (problem solving, case study, discussion)/didactic games/ distance learning*

*Laboratory classes: designing and conducting experiments*

Classes: working in groups and individual - task solving and proving theorems.

4. Assessment techniques and criteria

4.1 Methods of evaluating learning outcomes

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| --- | --- | --- |
| Learning outcome | Methods of assessment of learning outcomes (e.g. test, oral exam, written exam, project, report, observation during classes) | Learning format (lectures, classes,…) |
| LO-01 | exam, observation during classes | class |
| LO\_02 | exam, observation during classes | class |
| LO\_03 | exam, observation during classes | class |
| LO\_o4 | exam, observation during classes | class |
| LO\_05 | test, exam, observation during classes | class |
| LO\_06 | test, exam, observation during classes | class |
| LO\_07 | test, exam, observation during classes | class |
| LO\_08 | test, exam, observation during classes | class |
| LO\_09 | test, exam, observation during classes | class |
| LO\_10 | test, exam, observation during classes | class |
| LO\_11 | test, exam, observation during classes | class |
| LO\_12 | test, exam, observation during classes | class |
| LO\_13 | test, exam, observation during classes | class |
| LO\_14 | observation during classes | class |
| LO\_15 | observation during classes | class |
| LO\_16 | observation during classes | class |

4.2 Course assessment criteria

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| students are Assessed regularly solving tasks writing.  The examination of students' knowledge in an oral form.  Grading score:  3.0 for 50 - 60%, 3.5 for 61 - 70 %, 4.0 for 71 – 80%, 4.5 for 81 – 90%; 5.0 for 91 – 100 % |

5. Total student workload needed to achieve the intended learning outcomes

– number of hours and ECTS credits

|  |  |
| --- | --- |
| Activity | Number of hours |
| Scheduled course contact hours | 60 (30+30) |
| Non-contact hours - student's own work (preparation for classes or examinations, projects, etc.) | 140 (70+70) |
| Total number of hours | 200 (100+100) |
| Total number of ECTS credits | 10 (5+5) ECTS |

\* One ECTS point corresponds to 25-30 hours of total student workload

6. Internships related to the course/module

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| --- | --- |
| Number of hours |  |
| Internship regulations and procedures |  |

7. Instructional materials

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| Compulsory literature:   1. R. A. Sharipov, Course of linear algebra and multidimensional geometry, / Publ. of Bashkir State University — Ufa, 1996. 2. R. Shafarevich, A. Remizov, Linear algebra and geometry, Springer-Verlag Berlin Heidelberg, 2013. 3. V. V. Konev, Linear algebra, vector algebra and analytical geometry, Tomsk Polytechnic University, 2009 http://portal.tpu.ru:7777/SHARED/k/KONVAL/Textbooks/Tab1/Konev-Linear\_Algebra\_Vector\_Algebra\_and\_Analytical\_Geome.pdf |
| Complementary literature:   1. K. Nomizu, Fundamentals of Linear Algebra, McGrow-Hill, Inc., New York 1966. 2. G. Birkhoff, T.C. Bartee, Modern Applied Algebra, McGrow-Hill Book Company, New York 1980. |

Approved by the Head of the Department or an authorised person