**SYLLABUS**

**regarding the qualification cycle FROM 2023TO 2024**

1. Basic Course/Module Information

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| Course/Module title | Calculus ( I and II) |
| Course/Module code \* |  |
| Faculty (name of the unit offering the field of study) | *College of Natural Sciences* |
| Name of the unit running the course | *Institute of Mathematics* |
| Field of study | Mathematics |
| Qualification level | First degree |
| Profile | *Academic* |
| Study mode | *Full-time* |
| Year and semester of studies | *1 Year, 1 and 2 semester* |
| Course type | *Basic* |
| Language of instruction | English |
| Coordinator | Ewa Rak, PhD |
| Course instructor | *Stanisława Kanas, PhD, DSc* |

\* - as agreed at the faculty

1.1.Learning format – number of hours and ECTS credits

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Semester  (n0.) | Lectures | Classes | Colloquia | Lab classes | Seminars | Practical classes | Internships | others | **ECTS credits** |
| 1 |  | 30 |  |  |  |  |  |  | 5 |
| 2 |  | 30 |  |  |  |  |  |  | 5 |

1.2. Course delivery methods

☒ conducted in a traditional way

☒ involving distance education methods and techniques

1.3. Course/Module assessment (exam, pass with a grade, pass without a grade)

Exam

2. Prerequisites

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| Basic knowledge of mathematics on secondary school level, Secondary-school certificate |

3. Objectives, Learning Outcomes, Course Content, and Instructional Methods

3.1. Course/Module objectives

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| O1 | To familiarize students with axioms, construction and properties of a set of real numbers, with the concept of lower and upper bounds (infimum and supremum) as well as with the concept of function and basic properties of functions. |
| O2 | To familiarize students with definitions, examples and theorems regarding sequences and methods of examining the convergence of sequences. |
| O3 | To familiarize students with the basics of the theory of real functions of one variable - with the limits of functions, with the continuity and differentiability of functions, and with the use of the derivative of function to study the course of function variation. |
| O4 | To familiarize students with integrals: indefinite integral (primary function) and methods of its calculation, with the Riemann integral of one variable's real function and its applications in geometry and physics as well as with improper integral. |
| O5 | To familiarize students with definitions, examples and theorems regarding series, convergence criteria and methods for testing convergence of numerical series. |
| O6 | To familiarize students with functional sequences and series (including power series and Fourier series) and with the convergence criteria (point and uniform) of functional series. |

3.2. Course/Module Learning Outcomes (to be completed by the coordinator)

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| Learning Outcome | The description of the learning outcome  defined for the course/module | Relation to the degree programme outcomes |
| LO\_01 | has knowledge of the characteristic properties of the set of real numbers; its subsets and sequences of real numbers; supremum and infimum and mathematical induction;  knows the definition of a function; can study the basic properties of functions; can put together functions and determine inverse functions; | K\_W01; K\_ W02; K\_W03; K\_W04; K\_U01; K\_U02; K\_U04; K\_K01; K\_K02 |
| LO\_02 | knows the notion of sequence and the basic concepts associated with it. Knows the basic theorems regarding sequence convergence and their proofs;  knows the algebraic operations on sequences and their limits; knows the definition of the Euler number e (as the limit of a certain sequence);can prove the convergence of elementary sequences and calculate their limits; | K\_W02; K\_W03; K\_W04; K\_U01; K\_U02; K\_U04; K\_K01; K\_K02 |
| LO\_o3 | knows the definition of the limit of a function; knows the properties of limits of functions and fundamental theorems on limits of functions. Knows how to calculate limits of elementary functions; | K\_W02; K\_W03; K\_W04; K\_U01; K\_U02; K\_K01; K\_K02 |
| LO\_04 | knows the definitions of continuity of function (Heine and Cauchy); knows that algebraic operations on continuous functions lead to continuous functions; knows the theorem on the continuity of the inverse and complex functions; knows the properties of a continuous function on a connected set and on a compact set; can prove the continuity of elementary functions; | K\_W01; K\_W02; K\_W03; K\_W04; K\_U01; K\_U02; K\_K01; K\_K02 |
| LO\_05 | knows the concept of derivative of a function and properties of differentiable functions;  is able to study the differentiability of elementary functions; can calculate derivatives of first and higher orders; knows the mean value theorems and their proofs and consequences; is able to use differential calculus to study derivative, geometric interpretation; differentiation rules, extremum problems; convexity and concavity, application to graphing; knows the rule de l'Hospital and can use it to calculate the limits of functions; | K\_W01; K\_W02; K\_W03; K\_W04; K\_U01; K\_U02; K\_U04; K\_U05; K\_K01; K\_K02 |
| LO\_06 | knows the basic methods of calculating indefinite integrals (by parts and by substitution); is able to calculate indefinite integrals of rational, irrational and trigonometric functions; | K\_W01; K\_W02; K\_W03; K\_W04; K\_U01; K\_U02; K\_U06; K\_K01; K\_K02 |
| LO\_07 | knows the definition and basic properties of the Riemann integral; knows the basic theorems on the integrity of functions in the Riemann sense and their proofs; knows the relationship between the Riemann integral and indefinite integral (primary function);  is able to use the Riemann integral to solve geometric and physic problems; knows the definitions of improper integrals of various types and their basic properties. Knows the convergence criteria of improper integrals and can apply them; knows the most important examples of improper integrals and their applications; | K\_W01; K\_W02; K\_W03; K\_W04; K\_W07; K\_U01; K\_U02; K\_U06; K\_K01; K\_K02 |
| LO\_08 | knows the definition of the number series and the basic concepts associated with it; knows the criteria of convergence of series and knows how to use them to study the convergence of different series (positive and arbitrary). knows when changing the order in a series has no effect on its convergence and sum; | K\_W01; K\_W02; K\_W03; K\_W04; K\_W07; K\_U01; K\_K01; K\_K02 |
| LO\_09 | knows the definitions of point and uniform convergence of function sequences and series; knows the basic theorems about the continuity, integrity and differentiability of the functional sequence and the sum of the functional series, their proofs; knows the criteria for convergence of functional series and knows how to apply them; knows the definition and properties of a power series; known theorem on the differentiability and the integrality of power series;  knows the definition of Fourier series; knows the theorem on point convergence of the Fourier series; | K\_W01; K\_W02; K\_W03; K\_W04; K\_U01; K\_U02; K\_U05; K\_U06; K\_K01; K\_K02 |
| LO\_10 | can formulate questions to better understand the concepts, examples and theorems (and their proofs) in the field of differential calculus and express their own opinions on its basic issues; | K\_W01; K\_W02; K\_W03; K\_W04; K\_U01; K\_Uo2;K\_U05; K\_K02; K\_K06 |
| LO\_11 | finds applications of differential calculus in various areas of life and knowledge; | K\_Wo1; K\_W02; K\_W07; K\_U01; K\_U06; K\_K05 |
| LO\_12 | knows the limitations of his own knowledge and own abilities; understands the need for further education; independently searches in the literature and on the Internet for information on calculus. | K\_K01; K\_K02; K\_K03; K\_K05 |

**3.3. Course content (to be completed by the coordinator)**

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| Content outline |
| Set of real numbers. Axioms.  Basic properties of subsets of real numbers. Concept of supremum and infimum of a sets. Mathematical induction.  Sequences, boundedness, monotonicity, convergence and divergence of a sequences, limit, indeterminate forms.  Number series, basic definitions, sum of series. Convergence and divergence tests, absolute and conditional convergence tests. |
| Concepts of function, Limits and continuity. Continuity on a bounded segment. Asymptotes.  Derivative, geometric interpretation. Differentiation rules, extremum problems.  Convexity and concavity, application to graphing.  L'Hôspital's Rule, applications. |
| Indefinite integrals, introduction and basis properties.  Techniques of integrations (integration by substitution, by parts, by partial fractions, integration of trigonometric and exponential functions).  The Riemann integral, mean value theorem for integrals, fundamental theorem of calculus.  Numerical methods of integration, application of integrals to geometry and science. |
| Sequences and series of functions. Power series, Fourier series. Pointwise and uniform convergence. The Stone-Weierstrass theorem, the radius of convergence. Divergence of series. Expansion in power and Fourier series. Applications. |

3.4. Methods of Instruction

e.g.

*Classes: text analysis and discussion/project work (research project, implementation project, practical project)/ group work (problem solving, case study, discussion)/didactic games/ distance learning*

*Laboratory classes: designing and conducting experiments*

classes: working in groups and individual - task solving and proving theorems.

4. Assessment techniques and criteria

4.1 Methods of evaluating learning outcomes

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| Learning outcome | Methods of assessment of learning outcomes (e.g. test, oral exam, written exam, project, report, observation during classes) | Learning format (lectures, classes,…) |
| LO-01 | exam, observation during classes | class |
| LO\_02 | exam, observation during classes | class |
| LO\_03 | exam, observation during classes | class |
| LO\_o4 | exam, observation during classes | class |
| LO\_05 | test, exam, observation during classes | class |
| LO\_06 | test, exam, observation during classes | class |
| LO\_07 | test, exam, observation during classes | class |
| LO\_08 | test, exam, observation during classes | class |
| LO\_09 | test, exam, observation during classes | class |
| LO\_10 | observation during classes | class |
| LO\_11 | observation during classes | class |
| LO\_12 | observation during classes | class |

4.2 Course assessment criteria

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| students are Assessed regularly solving tasks writing.  The examination of students' knowledge in the oral form.  Grading score:  3.0 for 50 - 60%, 3.5 for 61 - 70 %, 4.0 for 71 – 80%, 4.5 for 81 – 90%; 5.0 for 91 – 100 % |

5. Total student workload needed to achieve the intended learning outcomes

– number of hours and ECTS credits

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| --- | --- |
| Activity | Number of hours |
| Scheduled course contact hours | 60 (30+30) |
| Non-contact hours - student's own work (preparation for classes or examinations, projects, etc.) | 140 (70+70) |
| Total number of hours | 200 (100+100) |
| Total number of ECTS credits | 10 (5+5) ECTS |

\* One ECTS point corresponds to 25-30 hours of total student workload

6. Internships related to the course/module

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| Number of hours |  |
| Internship regulations and procedures |  |

7. Instructional materials

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| Compulsory literature:   1. H. Jerome Keisler, Elementary Calculus, An Infinitesimal Approach. Second Edition, University of Wisconsin, 2012. 2. Raz Kupferman, Lecture Notes in Calculus, The Hebrew University, Jerusalem 2013. 3. David Guichard, Calculus, San Francisco, California, USA 2011. 4. Michael M. Dougherty and John Gieringe, First Year Calculus For Students of Mathematics and Related Disciplines, USA 2012. |
| Complementary literature:   1. Sean Mauch, Introduction to Methods of Applied Mathematics or Advanced Mathematical Methods for Scientists and Engineers, 2004 http://www.its.caltech.edu/~sean 2. J. Callahan, K. Hoffmann, D. Cox, Donald O. Shea, H. Pollatsek, L. Senechall, Calculus in context, New York University, USA 2008. |

Approved by the Head of the Department or an authorised person